**Definition:** If \( f(x) \) becomes closer and closer to a single number \( L \) as \( x \) gets closer and closer to \( c \) from either side, then
\[
\lim_{x \to c} f(x) = L,
\]
which is read as “the limit of \( f(x) \) as \( x \) approaches \( c \) is \( L \).”

**Examples:**

(a) \( \lim_{x \to -1} 3x^2 + 5 = 3 \times (-1)^2 + 5 = 8 \).

(b) \( \lim_{x \to 2} \sqrt{x^2 + 1} = \sqrt{2^2 + 1} = 5 \)

(c) \( \lim_{x \to 0} \frac{1}{x^2 + 1} = \frac{1}{0^2 + 1} = 1 \).

(d) Suppose that
\[
f(x) = \begin{cases} 
|x| & x \neq 0 \\
1 & x = 0
\end{cases}
\]
Then
\[
\lim_{x \to 0} f(x) = 0.
\]

**Note:**
\[
\lim_{x \to c} f(x)
\]
relies on the values of \( f(x) \) at \( x \) near \( c \), but may not have any connection to the value of \( f(x) \) at \( x = c \).

**Replacement Theorem:** If \( f(x) \) and \( g(x) \) are equal at all points except for at \( x = c \). Then,
\[
\lim_{x \to c} f(x) = \lim_{x \to c} g(x)
\]

**Example:**
\[
\frac{\sqrt{x + 1} - 1}{x} = \frac{\sqrt{x + 1} - 1}{x} \times \frac{\sqrt{x + 1} + 1}{\sqrt{x + 1} + 1} = \frac{x + 1 - 1}{x(\sqrt{x + 1} + 1)} = \frac{1}{\sqrt{x + 1} + 1}
\]
as long as \( x \neq 0 \) (why?). Therefore,
\[
\lim_{x \to 0} \frac{\sqrt{x + 1} - 1}{x} = \lim_{x \to 0} \frac{1}{\sqrt{x + 1} + 1} = \frac{\lim_{x \to 0} 1}{\lim_{x \to 0} (\sqrt{x + 1} + 1)} = \frac{1}{2}
\]

**Extra Problems:** Find the following limits

(a) \( \lim_{x \to 0} \frac{x^4 + 3x^3 - 5x^2}{x} \),

(b) \( \lim_{x \to 1} \frac{x^2 - 1}{x - 1} \)

(c) \( \lim_{x \to 1} \frac{x - 1}{x^2 - 1} \)

(d) \( \lim_{x \to 1} \frac{x - 1}{5x - 5} \)