Problem 5
(a) [6 points] The surface area of a cube is increasing at a rate of 4 in²/sec. How fast is the volume changing when the length of each edge is 2 inches long?

\[ A = 6a^2 \Rightarrow \frac{dA}{dt} = 12a \frac{da}{dt} \]

and we know that \( \frac{dA}{dt} = 4 \)

\[ V = a^3 \Rightarrow \frac{dV}{dt} = 3a^2 \frac{da}{dt} \]

Since \( 12a \frac{da}{dt} = 4 \)

\[ \Rightarrow a = 2, \quad \frac{da}{dt} = \frac{4}{12 \cdot 2} = \frac{1}{6} \text{ in/sec} \]

\[ \Rightarrow \frac{dV}{dt} = 3 \cdot 4 \cdot \frac{1}{6} = 2 \text{ in}^3/\text{sec} \]

(b) [6 points] Find the equation for the line that pass through the point (0, 0) and is parallel to the tangent of \( f(x) = x^2 \) at \( x = 1/2 \).

\[ f'(x) = 2x \quad a \cdot x = \frac{1}{2}, \quad f'(\frac{1}{2}) = 2 \cdot \frac{1}{2} = 1 \]

The line that passes through (0, 0) and has slope \( m = 1 \) is

\[ y - 0 = 1 (x - 0) \Rightarrow y = x \]
Extra Credit Problems-- Each of the following problems is optional and each is worth 10 points.

1.) Consider the function $f$ whose derivative $f'$ is given in the graph below. I repeat, the graph below is that of $f'$, NOT $f$. However, answer the following questions about $f$, NOT $f'$.
   a.) List the x-value(s) for which $f$ has a relative maximum.
       \[-7, \quad 1\]
   b.) List the x-value(s) for which $f$ has a relative minimum.
       \[-3, \quad 7.5\]
   c.) List the x-value(s) for which $f$ has an inflection point.
       \[-5, \quad -1, \quad 3\]

2.) A square is inscribed in the given right triangle. Find the area of the square.

\[
A = \sqrt{13^2 - 12^2} = 5
\]

Using triangles $ABC$ and $ADE$, we have:

\[
\frac{x}{12-x} = \frac{5}{12} \quad (= \tan \theta)
\]

\[
\Rightarrow 12x = 60 - 5x \quad \Rightarrow x = \frac{60}{17}
\]

\[
\Rightarrow \text{Area} = \left(\frac{60}{17}\right)^2
\]