Part I:
Background and Geometric Methods

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Overview

- Foundations
  - Time-Varying Vector Fields
  - Numerical Integration

- Geometric Methods
  - Integral Curves
  - Integral Surfaces
  - Advanced Geometric Methods
A time-varying vector field is a map

\[ \vec{v} : I \times D \rightarrow \mathbb{R}^n \quad I \subseteq \mathbb{R}, \quad D \subseteq \mathbb{R}^n \]

An associated ordinary differential equation (ODE)

\[ \frac{dx}{dt} = \vec{v}(t, x) \]

with initial condition

\[ x(t_0) = x_0 \]

is called integral curve, trajectory or orbit.
Existence and uniqueness of integral curves if and only if $\vec{v}$ is continuous in $t$ and Lipschitz in $x$:

$$\|\vec{v}(t, x) - \vec{v}(t, y)\| < L \|x - y\| \quad \forall x, y \in D, t \in I$$

Holds for virtually all discrete vector fields.
Numerical Integration

- Closed form solutions (analytic) to integral curve ODE are not available in general
  - Numerical approximation / ODE solvers

- ODE solvers come in many shapes and sizes
  - vector field properties, approximation properties, ...
    - single-step (for visualization) vs. multi-step
Numerical Integration

Euler scheme: simplest method, fast, inaccurate, unstable

\[ x_{n+1} = x_n + \Delta t \vec{v}(x_n, t_n) + O((\Delta t)^2) \]
Numerical Integration

Runge-Kutta, 2nd order: try Euler step, then improve

\[ \vec{k}_1 = \vec{v}(x_n, t_n) \]
\[ \vec{k}_2 = \vec{v}(x_n + \frac{1}{2}\Delta t \vec{k}_1, t_n + \frac{1}{2}\Delta t) \]

\[ x_{n+1} = x_n + \Delta t \vec{k}_2 + O((\Delta t)^3) \]
Numerical Integration

- Runge-Kutta schemes exist for arbitrary orders
  - 3rd (3 stages), 4th (4 stages), 5th (6 stages), ...

- Adaptive stepsize control
  - chooses $\Delta t$ based on observed vector field complexity

- Popular schemes: Runge-Kutta-Fehlberg (RKF / RK54)
  - six stages give order 4 and order 5 approximation
    - comparison allows error estimate, increase or decrease in step size in response
Numerical Integration

- Integration efficiency depends on lookups of $\vec{v}$:
  - e.g. large regular mesh representation: expensive
  - e.g. unstructured mesh representation: very expensive
    - requires spatial data structures (e.g. kd-tree [Langbein 2003])

- Higher approximation order $\rightarrow$ fewer steps in practice
- Adaptive stepsize control is the key to efficiency

(different rules apply e.g. for GPUs, see also Part IV)
Numerical Integration

- Visualization requires graphical output... so we must constrain $\Delta t$ to guarantee good graphics

- Not very good for efficiency...
- Better: methods with dense output, polynomial per step
  - sample arbitrary integral curve points after integration
  - proven scheme: Dormand-Prince (DOPRI5) [Prince 1981]
Numerical Integration

Caveat for time-varying vector fields:

- Integration might span multiple time steps
- Even in medium sized data, all time steps do not fit into main memory

- Even for multiple curves, traverse the dataset only once:
  - integrate all curves over current time interval, load next interval, repeat
  - streaming approach
Direct Visualization of integral curves: pathlines

Invoke intuition about trajectories of advected particles.

“Fishtank” dataset courtesy Paul Fischer,
Integral Curve Visualization

Direct Visualization of integral curves: particles

Animation conveys vector field dynamics through particle speed.

Data courtesy Jürgen Schneider, AVL GmbH
Integral Curve Visualization

Direct Visualization of integral curves: pathlets

Particle speed encoded in pathlet length.

Data courtesy Jürgen Schneider, AVL GmbH
Integral Curve Visualization

• Direct Integral Curve Visualization
  • perceptually, works well for few lines but cluttered for many
  • good in exploratory settings
    • specify seed set, see curves or particles, repeat
  • Drawback: does not mimic physical experiment
    • particle trajectories usually not observable
Integral Curve Visualization

Physical Experiment
Délery, 2005
Integral Curve Visualization

- Time Lines: mimic “line of smoke or dye”

Set of adjacently seeded particles at an instant of time
Integral Curve Visualization

- Streak Lines: mimic flow of smoke or dye from a nozzle

Set of all fluid particles that have gone through a common point at some time in the past.
Integral Curve Visualization

Time Lines vs. Streak Lines

- Timelines
- Streaklines
Integral Curve Visualization

Generalized Streak Lines:
moving seed point

Set of all particles that pass through a space-time curve.
Integral Surfaces

- Integral Surfaces address the problem of clutter with many line-type primitives
- Idea: exploit coherent movement of neighboring particles and connect them to surfaces
- Surfaces defined by family of integral curves
Integral Surfaces

Path surface: surface spanned by all trajectories of a one-dimensional family of particles
Path Surfaces

Path Surface Example

seeding curve

Data courtesy Markus Rütten, DLR Germany
Integral Surfaces

• Benefits of surfaces over curves:
  • Shape through lighting
  • Depth perception
  • Coherent motion

But: integral surfaces are much more expensive to compute:
• typically 10x for path surfaces
• typically 100x – 1000x for time and streak surfaces
Integral Surfaces

- Path surface approximation, in a nutshell: Approximate surface as a series of timelines

[Garth et al., 2008]
Integral Surfaces

- Path surface approximation, in a nutshell:
  Discretize time lines by a finite number of path lines

[Garth et al., 2008]
Integral Surfaces

- Path surface approximation, in a nutshell: Keep time lines well approximated by refinement

[Garth et al., 2008]
Integral Surfaces

- Path surface approximation, in a nutshell:
  Keep time lines well approximated by refinement

[Garth et al., 2008]
Integral Surfaces

- Nice property of this algorithm: Still only need to traverse dataset only once

[Garth et al., 2008]
Streak Surfaces

Similar approach for time and streak surfaces using triangle meshes

[Krishnan et al., 2009]
Integral Surfaces

Exact approximation can be costly: many pathlines must be integrated.

Approximate method: Smoke Surfaces

- Start with fine approximation of time or streak surface
- As the approximation (triangulation) degenerates, smoothly fade it out
- Transparency term is physically inspired, mimics appearance of smoke
Integral Surfaces

For small data:

• [Bürger et al. 2009]
  Point-based/Quad-based streak surfaces + GPU

• [Weiskopf et al. 2004]
  Point-based path surfaces + GPU

• [Westermann et al. 2000]
  Time-Surfaces as level sets, on the GPU
Non-GPU / Large Data / High Accuracy: Surface computation is not interactive.

Using dense integration (DOPRI5), the full surface evolution can be stored as:
- a set of integral curves / trajectories
- a set of changes to the approximation structure

Integral Surfaces

- Non-GPU / Large Data / High Accuracy: Surface computation is not interactive
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Integral Surfaces

- Reconstruction of the surface at any point in its evolution:
  - sample trajectories at desired time
  - apply all mesh changes up to that time

- Storing only trajectories and mesh changes yields a very compact representation
  - much better than storing entire surface per timestep

- Enables interactive viewing + navigation in time
  - get the most out of non-interactive computation
Integral Surfaces

- Example: Jet flow
- Dataset: 36GB
- Streak surface: 250MB at high accuracy
Integral Surface Visualization

Streak surface rendering, with nice transparency and lighting can be done in real time.
Integral Surface Visualization

Is there a way to convey which parts of the surfaces where seeded simultaneously, or at the same location (path lines, time lines and streak lines)?
Integral surfaces yield themselves well to texture mapping (going back to [Löffelmann et al. 1997]).
Textured surfaces combine the advantages of integral lines and surfaces:

Integral Surface (path, time, streak surface)
- shape, depth
- no clutter

Texture (path-, time, streak lines)
- improved temporal and spatial orientation
Integral Surface Visualization

Vortex formation behind an ellipsoidal body

Data courtesy Markus Rütten, DLR Germany
Integral Surface Visualization

Data courtesy Markus Rütten, DLR Germany
Integral Surface Visualization

Data courtesy Markus Rütten, DLR Germany
More Geometric Techniques

• Eyelet Surfaces: stationary visualization of instationary flow, surface spanned by all pathlines passing a common seed point [Wiebel et al. 2006]

• Pathline Predicates: define analytic criteria to group pathlines into sets [Salzbrunn et al. 2007]

• Geometric verification of features: compare behavior of integral curves or surfaces against expected features [Jiang et al. 2002, Garth et al. 2004]

• Many more, see references...
References


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